

# Slowly Rotating Dilaton Black hole In Anti-de Sitter Spacetime

Tanwi Ghosh\* and Soumitra SenGupta†

*Department of Theoretical Physics,  
Indian Association for the Cultivation of Science,  
Jadavpur, Calcutta - 700 032, India*

Rotating dilaton black hole solution for asymptotically anti-de Sitter spacetime are obtained in the small angular momentum limit with an appropriate combination of three Liouville-type dilaton potentials. The angular momentum, magnetic dipole moment and the gyromagnetic ratio of such a black hole are determined for arbitrary values of the dilaton-electromagnetic coupling parameter. .

## I. INTRODUCTION:

Scalar coupled black hole solutions with different asymptotic spacetime structure is a subject of interest for a long time. There has been a renewed interest in such studies ever since new black hole solutions have been found in the context of string theory. The low energy effective action of string theory contains two massless scalars namely dilaton and axion. The dilaton field couples in a non-trivial way to other fields such as gauge fields and results into interesting solutions for the background spacetime [1]. It is found that the dilaton changes the causal structure of spacetime and leads to curvature singularities at finite radii. Subsequently various other properties have also been explored for such scalar coupled black holes [2, 3, 4, 5, 6, 7, 8, 9]. Most of these scalar coupled black hole solutions however were asymptotically flat. Gao and Zhang [10] obtained the asymptotically non-flat charged dilaton black hole solutions in anti-de Sitter and de Sitter metric in Schwarzschild coordinate using the combination of three Liouville type dilaton potentials. Such potential may arise from the compactification of a higher dimensional supergravity model [11, 12] which originates from the low energy limit of a background string theory.

At this time, there was also a growing interest to study the rotating black hole solutions in presence of dilaton coupled electromagnetic field in the background. An exact solution for a rotating black hole with a special dilaton coupling was derived using the inverse scattering method [13]. In an alternative approach Horne and Horowitz developed a simpler perturbative technique to find such rotating black hole solution where a small angular momentum is introduced as a perturbation in a non-rotating system in presence of a general dilaton-electromagnetic coupling. They studied the properties of asymptotically flat charged dilaton black holes in the limit of infinitesimally small angular momentum [14, 15]. A Similar work then followed by introducing an infinitesimal small charge to the black hole[16]. Following the method adopted in [14], asymptotically non-flat rotating dilaton black holes are obtained in [17] for small value of the angular momentum. In a different context M.H. Dehghani et al obtained charged rotating dilaton black string solutions [18]. However charged rotating dilaton black hole solutions for an arbitrary dilaton-electromagnetic coupling parameter in a de sitter or anti-de Sitter spacetime is yet to be found.

The interest for studying such dilaton black holes with non-vanishing cosmological constant has several reasons. Gauged supergravity theories in various dimensions are obtained with negative cosmological constant in a supersymmetric theory. In such scenario anti-de Sitter spacetime constitutes the vacuum state and the black hole solution in such a spacetime becomes an important area to study[19, 20, 21, 22, 23, 24]. Apart from the general interest as a classical solutions of field equations in the framework of general relativity, there are also cosmological implications of exploring the features of anti-de sitter and de sitter spacetime [25, 26, 27, 28, 29]. The behaviour of galaxies away from their centres suggests the presence of a very large amount of dark matter with  $\frac{1}{r^2}$  distribution of energy. One can imagine this dark matter in the form of a scalar field such as a dilaton. Asymptotically flat behaviour is not consistent with the observed non-decaying velocity profile and therefore the Ads or ds solutions are of considerable interest in this context. Moreover the observed rotational motion in galaxies further suggests the need of studying the rotating metric in such a spacetime.

In the backdrop of the scenarios described so far it is therefore worthwhile to study the rotating black hole solutions in a spacetime with non-zero cosmological constant in presence of dilaton-electromagnetic coupling. Exponential potentials for scalar fields have been used by many workers in this context, which closely resembles to Gao-Zhang's model. We here study various features of a rotating charged dilatonic black hole solution in a space with non-zero cosmological constant. In particular we concentrate on the anti-de sitter spacetime as considered in [10]. Introducing a small angular momentum, following Horne and Horowitz, we aim here to obtain a rotating black hole solution in anti-de Sitter spacetime by generalizing Gao and Zhang's work. We then determine the expressions for angular

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\* E-mail: tanwi.ghosh@yahoo.co.in

† E-mail: tpssg@iacs.res.in

momentum, magnetic dipole moment and the gyromagnetic ratio for such a black hole.

## II. FIELD EQUATIONS AND SOLUTIONS:

We begin with an action of the form,

$$S = \int d^4x \sqrt{-g} [R - 2\partial_\mu \varphi \partial^\mu \varphi - V(\varphi) - e^{-2\alpha\varphi} F^2] \quad (1)$$

where  $R$  is the scalar curvature,  $F^2 = F_{\mu\nu} F^{\mu\nu}$ ,  $F_{\mu\nu}$  being the usual Maxwell field tensor,  $V(\varphi)$  is the potential for the dilaton field  $\varphi$  and  $\alpha$  measures the strength of the coupling between the electromagnetic field and  $\varphi$ . While  $\alpha = 0$  corresponds to the usual Einstein-Maxwell-scalar theory,  $\alpha = 1$  indicates the dilaton-electromagnetic coupling that appears in the low energy string action in Einstein's frame. Varying the action w.r.t the metric, the Maxwell field and dilaton field respectively we obtain the equations of motion as,

$$R_{ab} = 2\partial_a \varphi \partial_b \varphi + \frac{1}{2} g_{ab} V + 2e^{-2\alpha\varphi} [F_{ac} F_b^c - \frac{1}{2} g_{ab} F_{cd} F^{cd}] \quad (2)$$

$$\partial_a [\sqrt{-g}] e^{-2\alpha\varphi} F^{ab} = 0 \quad (3)$$

$$\nabla^2 \varphi = \frac{1}{4} \frac{\partial V}{\partial \varphi} - \frac{\alpha}{2} e^{-2\alpha\varphi} F_{cd} F^{cd} \quad (4)$$

For arbitrary value of  $\alpha$  in anti-de Sitter space the form of the dilaton potential is chosen as [10],

$$V(\varphi) = \frac{2}{3} \lambda \frac{1}{(\alpha^2 + 1)^2} [\alpha^2 (3\alpha^2 - 1) e^{-2\varphi/\alpha} + (3 - \alpha^2) e^{2\alpha\varphi} + 8\alpha^2 e^{\alpha\varphi - \varphi/\alpha}] \quad (5)$$

with the metric to be of the form [10],

$$\begin{aligned} ds^2 = & -[(1 - \frac{r_+}{r})(1 - \frac{r_-}{r})^{\frac{(1-\alpha^2)}{(1+\alpha^2)}} - \frac{1}{3} \lambda r^2 (1 - \frac{r_-}{r})^{\frac{2\alpha^2}{(\alpha^2+1)}}] dt^2 \\ & + [(1 - \frac{r_+}{r})(1 - \frac{r_-}{r})^{\frac{(1-\alpha^2)}{(1+\alpha^2)}} - \frac{1}{3} \lambda r^2 (1 - \frac{r_-}{r})^{\frac{2\alpha^2}{(\alpha^2+1)}}]^{-1} dr^2 \\ & + r^2 (1 - \frac{r_-}{r})^{\frac{2\alpha^2}{(\alpha^2+1)}} d\Omega^2 - 2af(r) \sin^2 \theta dt d\phi \end{aligned} \quad (6)$$

Here  $r_+$  and  $r_-$  are respectively the event horizon and Cauchy horizon of the black hole. The parameter ' $a$ ' measures the angular momentum which we shall choose to be small. In other words it can be viewed as a small axisymmetric perturbation in an anti-de Sitter background. We now determine the form of  $f(r)$  for such an anti-de Sitter metric and look for the possible black hole structure in that spacetime.

Integrating the Maxwell's equations we obtain the first derivative of the scalar potential as,

$$-A'_0 = F_{01} = \frac{Qe^{2\alpha\varphi}}{h} \quad (7)$$

where  $Q$ , the integration constant is related to the physical electric charge as will be shown later and  $h$  for the anti-de Sitter metric is given by,

$$h = r^2 (1 - \frac{r_-}{r})^{\frac{2\alpha^2}{(\alpha^2+1)}} \quad (8)$$

It may be noted that the only term in the metric that changes to the order of the angular momentum parameter  $a$  is  $g_{t\phi}$ . It is easy to see that the dilaton solution does not change to the order of  $a$  and the solution for the  $\phi$  component of the vector potential in the presence of rotation is given by,

$$A_\phi = -aQc(r) \sin^2 \theta \quad (9)$$

where  $c(r)$  satisfies

$$Q(uc'e^{-2\alpha\varphi})' - \frac{2Qce^{-2\alpha\varphi}}{h} = -Q(\frac{f}{h})' \quad (10)$$

with,

$$u = [(1 - \frac{r_+}{r})(1 - \frac{r_-}{r})^{\frac{(1-\alpha^2)}{(1+\alpha^2)}} - \frac{1}{3}\lambda r^2(1 - \frac{r_-}{r})^{\frac{2\alpha^2}{(\alpha^2+1)}}]$$

For anti-de sitter spacetime the solution for the dilaton field is [10],

$$e^{2\alpha\varphi} = e^{2\alpha\varphi_0}(1 - \frac{r_-}{r})^{\frac{2\alpha^2}{(\alpha^2+1)}} \quad (11)$$

The remaining equation of motion which is needed to be satisfied for small  $a$  is,

$$\frac{f''}{f} = \frac{h''}{h} - \frac{4Q^2 c'}{fh} \quad (12)$$

For small value of  $a$ , using the ansatz  $c(r) = \frac{1}{r}$  [17, 30], the solution of  $f(r)$  from equation (12) and (15) turns out to be,

$$f(r) = \frac{r^2(1+\alpha^2)^2 e^{-2\alpha\varphi_0}}{(1-\alpha^2)(1-3\alpha^2)r_-^2} (1 - \frac{r_-}{r})^{\frac{2\alpha^2}{(\alpha^2+1)}} \quad (13)$$

$$-e^{-2\alpha\varphi_0}(1 - \frac{r_-}{r})^{\frac{(1-\alpha^2)}{(1+\alpha^2)}} [1 - \frac{r_+}{r} + \frac{(1-\alpha^2)}{(1+\alpha^2)} \frac{r}{r_-} + \frac{r^2(1+\alpha^2)^2}{(1-\alpha^2)(1-3\alpha^2)r_-^2}]$$

$$+ \frac{\lambda}{3} e^{-2\alpha\varphi_0} r^2 (1 - \frac{r_-}{r})^{\frac{2\alpha^2}{(\alpha^2+1)}} \quad (14)$$

It is straightforward to verify that in our case for  $\lambda = 0$ , the expression for  $f(r)$  agrees with the corresponding expressions for a slowly rotating dilaton black hole solutions in a flat spacetime as obtained in [14, 15]. For slowly rotating dilaton black hole, we learn from [14] that the surface gravity and the area of the event horizon do not change to the order of the angular momentum parameter  $a$ . In the linear approximation in rotation parameter  $a$  we obtain the expression for angular momentum for anti-de Sitter case using Komar approach [31]. For rotating AdS black hole, angular momentum and mass are defined by

$$M' = -\frac{1}{8\pi} \oint *d(\delta\widehat{\xi}_{(t)}) \quad (15)$$

and

$$J' = \frac{1}{16\pi} \oint *d(\delta\widehat{\xi}_{(\phi)}) \quad (16)$$

where  $*$  denotes the Hodge dual and  $\xi$  the Killing one form.  $(\delta\widehat{\xi})$  represents the difference between the Killing isometries of the spacetime and its background spacetime. Using the above two integrands in the asymptotic limit, we obtain,

$$\delta\xi_{(t)}^{t;r} = \frac{r_+}{2r^2} + (\frac{1-\alpha^2}{1+\alpha^2}) \frac{r_-}{2r^2} + O(\frac{1}{r^2}) \quad (17)$$

and

$$\delta\xi_{(\phi)}^{t;r} = e^{-2\alpha\varphi_0} \frac{r_+}{r^2} + e^{-2\alpha\varphi_0} \frac{r_-}{r^2} \frac{(3-\alpha^2)}{3(\alpha^2+1)} + O(\frac{1}{r^2}) \quad (18)$$

Integrating (19) and (20) over two sphere in  $r \rightarrow \infty$  limit we get

$$M' = \frac{r_+}{2} + (\frac{1-\alpha^2}{1+\alpha^2}) \frac{r_-}{2} \quad (19)$$

and

$$J' = \frac{ae^{-2\alpha\varphi_0}}{2} [r_+ + \frac{(3-\alpha^2)}{3(1+\alpha^2)} r_-] \quad (20)$$

For the anti-de Sitter dilatonic black hole the expression for actual mass can be written as [31]

$$M_{actual} = M' - a \frac{\lambda}{3} J' = M' - a^2 \frac{\lambda}{3} \frac{e^{-2\alpha\varphi_0}}{2} [r_+ + \frac{(3-\alpha^2)}{3(1+\alpha^2)} r_-] = M' \quad (21)$$

Here for slowly rotating black hole we have ignored the term of the order of  $a^2$ .

It is interesting to note that the expression for  $J$  is same as that obtained in [14]. Using the expression for  $h$  (from equation 8) and expression for  $e^{2\alpha\phi}$  ( from equation 11) in equation (7) and (9) one obtains the following expressions for the radial fields [30],

$$E_r = \frac{Qe^{2\alpha\varphi_0}}{r^2} \quad (22)$$

and

$$B_r = \frac{aQ\sin^2\theta}{r^2} \quad (23)$$

The Gaussian flux of electric field gives the value of electric charge as follows,

$$Q' = \frac{1}{4\pi} \oint *F = Qe^{2\alpha\varphi_0} \quad (24)$$

Magnetic dipole moment for this slowly rotating dilaton black hole in anti-de Sitter spacetime can be defined as

$$\mu' = Q'a = g \frac{Q'J'}{2M'} \quad (25)$$

Substituting  $M'$ ,  $J'$  and  $Q'$  from equ.(17), (18), (19) and (22),  $g$  can be obtained as

$$g = 2 - \frac{4\alpha^2 r_-}{[(3 - \alpha^2)r_- + 3(1 + \alpha^2)r_+]} \quad (26)$$

It may be noted that while using equ.(20) to estimate  $g$  above, we have ignored the term quadratic in the rotation parameter  $a$ . As a result in the linear approximation in  $a$ , the above expression for  $g$  turns out to be same as that found in [14]. Moreover it is easy to check that for  $\alpha = 0$ , we retrieve  $g=2$  and in the limit  $\alpha \rightarrow 1$  the value of  $g$  lies between 2 and  $3/2$ .

#### IV. CONCLUDING REMARKS:

Starting from the non-rotating charged dilaton black hole solutions in anti-de Sitter spacetime, we have obtained the solution for the rotating charged dilaton black hole by introducing a small angular momentum following the perturbative method suggested in [14]. Since it is well-known, that in the presence of one Liouville type dilaton potential, no de Sitter or anti-de Sitter dilaton black hole exists even in absence of rotation therefore considering the proposal in [10] of three Liouville type dilaton potential we show that such potential leads to slowly rotating charged dilaton black holes solutions in an anti-de Sitter spacetime. As expected, our solutions  $f(r)$  reduces to [14] for  $\lambda = 0$ . The forms of angular momentum  $J$  and gyromagnetic ratio  $g$  are obtained. Their values to the linear order in the angular momentum parameter  $a$  turn out to be same as that obtained in [14]. It is interesting to note that the dilaton field modifies the value of  $g$  from 2 through the coupling parameter  $\alpha$  which measures the strength of the dilaton-electromagnetic coupling.

As discussed earlier, the presence of such anti-de sitter dilatonic charged rotating black hole is inevitably associated with an accompanying scalar field with appropriate Liouville-type potential. Their study therefore may lead to a better understanding of the origin of the dark matter in the universe. This work brings out the corresponding spacetime solution along with the associated black hole structure in the limit of small angular momentum.

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